JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 17, No. 1, January–February 1994

Mixed Strategy Guidance: A New High-Performance Missile Guidance Law

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The paper describes a new guidance law design process for the terminal homing phase of a radar missile against a maneuvering target with electronic countermeasures (ECM) capability. The process is based on a zero-sum, imperfect information differential game formulation and the concept of mixed strategies. It is demonstrated that the resulting mixed strategy guidance (MSG) law has a much better performance than any known guidance law.

Nomenclature

a = lateral acceleration
E = operator of expectation
G = shaping filter transfer function
g = acceleration of gravity
J = cost function
K = guidance gain
k = shaping filter gain
N' = effective proportional navigation gain
n = load factor
P_d = probability of target destruction
P_{ik} = SSKP for a given strategy pair δ_{ei} and δ_{pk}
R = range
R_ℓ = characteristic warhead lethal radius
s = Laplace variable
t_R = time to roll for maneuver change, see Fig. 3
V = velocity
$\mathcal{V}_m$ = value of the game with mixed strategies, guaranteed SSKP value
w_j = amplitude of electronic jinking
$w_{\max}$ = semispan of target aircraft
α, β = probabilities (components of mixed strategies)
γ = shaping filter lead parameter
Δ_e = pure strategy set of the evader
Δ_p = pure strategy set of the pursuer
δ_{ei} = pure evader strategy
δ_{pk} = pure pursuer strategy
ζ = shaping filter damping coefficient
Θ = normalized time to go
θ = vertical flight-path angle, see Fig. 1
λ = line-of-sight angle
λ_t = random telegraph shaping filter parameter
μ = pursuer-evader maneuver ratio
ν = random phase angle
σ = standard deviation
τ = time constant
Φ = spectral density
ϕ_e = roll angle of the evader, see Fig. 2
ψ = heading angle, see Fig. 1
ω_e = frequency of periodical evader maneuver, see Fig. 3
ω_j = frequency of electronic jinking
$\tilde{\omega}$ = shaping filter frequency

Subscripts

c = critical value
E, e = evader
f = final value
gl = glint
i, k = running indices
j = electronic jinking
max = maximum value
P, p = pursuer
0 = initial value

I. Introduction

THE terminal phase of an encounter between a radar-guided missile and an aircraft can be formulated as a differential game with imperfect, noise-corrupted information. It has been shown that if the missile can obtain perfect measurements of the line-of-sight (LOS) rate, then there exists a guidance law, derived from linear differential game theory,[1,2] that reduces the maximum miss distance achievable by the optimal target maneuver to a negligibly small value. In practical terms it means that target destruction is guaranteed in spite of all feasible target maneuvers.

Since target detection in a radar-guided missile is noise corrupted, an estimator is incorporated in the guidance loop. The designer of the estimator has to make assumptions on the expected target maneuvers. The better these assumptions match the actual maneuver, the better the overall performance of the missile will be. This observation leads to the logical conclusion that the optimal strategy of the target is to maneuver randomly in the most unpredictable way. In other words, the optimal evader strategy is a mixed strategy. Nevertheless, all known efforts of missile design have been concentrated on finding a single optimal guidance law,[3-8] i.e., a "pure" optimal pursuer strategy.

Forte and Shinar[9] were the first to introduce a concept that allowed (in the frame of a zero-sum imperfect information differential game) both players to use a mixed strategy (a probability distribution over the set of pure strategies). The theoretical framework for computing the optimal mixed strategies was outlined in different levels of mathematical rigor and details,[10-12] whereas some application examples were separately presented.[13,14]

As the result of further investigation, a practical missile design approach, called the mixed strategy guidance (MSG), has been established and validated for a typical missile end-game scenario. The most important feature of this approach for guidance law design, based on the optimization against the worst target maneuver options, is the elimination of the "weak spots" in the missile performance envelope. This property remains valid even if the outcome of the synthesis is a single guidance law (a "pure" guidance strategy).

If no single guidance strategy is found to have satisfactory performance (which is mostly the case), the MSG approach

Received July 10, 1992; revision received Feb. 22, 1993; accepted for publication Feb. 25, 1993.

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suggests to design two (or more) pure guidance strategies, each having adequate performance against some subset of the set of all possible target behaviors (the pure strategy set of the target). The design objective is to optimize the performance against the worst target strategy of this subset. Nevertheless, a specific pure guidance strategy may have a poor performance against other subsets of target behavior. In an MSG design, a random selection is made between the different pure guidance strategies, with an optimal probability distribution, which maximizes the *guaranteed* (worst case) performance.

The objective of the present paper is to outline an actual MSG synthesis process for a three-dimensional scenario and to compare the performance of the MSG law with conventional guidance laws, such as proportional navigation (PN) and augmented proportional navigation (APN). Details, which are out of the scope of the present paper, can be found by interested readers in Refs. 9–14.

II. Problem Formulation

The detailed formulation of the terminal phase of an encounter between a radar-guided missile and a maneuvering aircraft as a two-person, zero-sum, imperfect information game, in which both players are allowed to use mixed strategies, was presented in previous works.[13,14] For the sake of completeness, an outline of this formulation is repeated here.

Terminology

A guidance law is understood to be a function that maps the estimated state into acceleration commands, regardless of the form of the estimator used in the guidance loop. The combination of a guidance law and an estimator will be referred to as a "pure guidance strategy." The missile and the target aircraft are also called the "pursuer" and "evader," respectively.

Scenario Description

The main elements of the assumed scenario are the following:

1) The encounter is three dimensional.

2) The game starts and takes place in the vicinity of the collision course (see Fig. 1) and terminates when the range rate becomes zero.

3) To enhance survivability, the target may make use of electronic countermeasures (ECM) simultaneously with its maneuver. The ECM technique considered in this work is "electronic jinking" (EJ).[15] It generates a deterministic motion of the aircraft's radar reflection center from wing tip to wing tip. Whenever EJ is applied, the inherent stochastical fluctuation of the aircraft radar reflection center, called the "glint" noise, becomes hardly observable.

4) For the sake of simplicity, it is assumed that the motion of the target is confined to the collision plane (assumed to be horizontal). Note that, even if this assumption is adopted and effects of gravity are neglected, the motion of the missile will be three dimensional because the disturbances caused by the EJ or the glint noise have a significant transversal component (see Fig. 2).

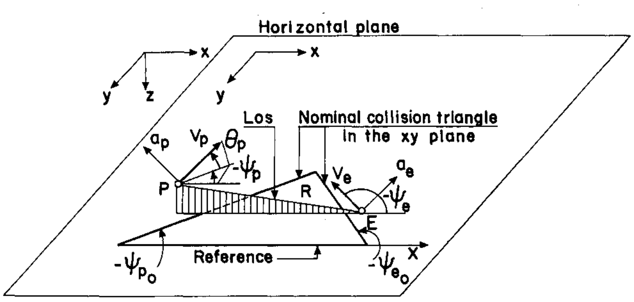

Fig. 1 Geometry of the end-game encounter.

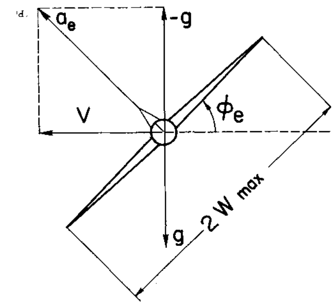

Fig. 2 Front view of a maneuvering target.

Information Structure

Throughout the duration of the game, the pursuer measures the relative range R, its rate of change $\dot{R}$, and the LOS angle λ, relative to an inertial reference line. It is assumed that the range and range rate measurements are exact and that the angle measurement is corrupted by noise. The range and range rate information is processed to give an accurate estimate of the "time to go," whereas the range and angle information yield a "noisy measurement" of the evader's relative position perpendicular to the reference line. The angular measurement is perturbed either by the glint noise or by the intentionally generated EJ.

The evader knows when the game starts but has neither measurements on the state of the game nor knowledge of the duration of the game.

Mixed Strategies

Mixed strategy is defined as a probability distribution over a set of pure strategies.[16] The pure strategy set of the pursuer Δ_p is defined as a countable set of guidance strategies δ_{pk} of a given structure. The pure strategy set of the evader Δ_e is defined as a (possibly infinite) set of actions δ_{ei}, each of which is composed of a maneuver sequence and some ECM (e.g., EJ).

The rules of the game are such that at the beginning of the game (or shortly before it) each player "selects" through a chance mechanism one of its pure strategies (the evader selects δ_{ei} with probability α_i and the pursuer selects δ_{pk} with probability β_k) and plays according to it until the end of the game. The chance mechanism is a realization of the mixed strategy for the respective player.

Evader Model

The evader is represented by a constant speed point-mass model for which the lateral acceleration options in the horizontal plane are given by $a_e = 0$ (nonmaneuvering target), $a_e = \pm g\, n_{\max}$ (constant target maneuver), and $a_e = g \tan \phi_e$ (periodical random phase maneuver with frequency ω_e). The maximum bank angle $(\phi_e)_{\max}$ is related to the maximum load factor $n_{\max}$ by the equation

$$\cos\left[(\phi_e)_{\max}\right] = 1/n_{\max} \tag{1}$$

The roll dynamic constraints of the aircraft are taken into account in the average sense. The time required to change the bank angle by the amount of $2(\phi_e)_{\max}$ is t_R (see Fig. 3).

The electronic jinking is also assumed to be periodical. It is given by

$$w_j = w_{\max}\, \mathrm{sign}\left[\sin(\omega_j t + \nu_j)\right] \tag{2}$$

The respective random phases of the kinematic maneuver and of the EJ are taken to be uncorrelated.

In the present investigation the pure strategy set of the evader Δ_e is composed of 90 elements, i.e., all of the combinations of 9 maneuver options consisting of a straight-flying evader, a constant evader maneuver ($\omega_e = 0$), and periodical random phase maneuvers at 7 different frequencies ($\omega_e = 0.25$, 0.50, 0.75, 1.0, 1.5, 2.0, and 3.0) and 10 ECM options consisting of a set of random phase EJ processes at 9 different frequencies ($\omega_j = 0$, 0.25, 0.50, 0.75, 1.0, 1.5, 2.0, 2.5, and 3.0) and the no-ECM option (NJ). (At $\omega_j = 0$ the aircraft radar reflection center is fixed at one of the wing tips.)

Pursuer Model

The pursuer is a radar-guided homing missile represented by a simplified mathematical model. The kinematics of the missile are of a constant speed point-mass model with limited lateral acceleration. Moreover, it is assumed that the three-dimensional interception is accomplished by two identical decoupled guidance channels operating in perpendicular planes. Each guidance channel model consists of the following elements:

1) The "seeker" reconstructs the LOS direction from radar measurements corrupted by glint-type noise and provides the relative lateral displacement with respect to a stabilized reference. The generation of this signal doesn't involve any delay.

2) The "estimator" extracts from the noisy signal of the seeker a smoothed estimate of the relative state.

3) The "guidance computer" determines the required lateral acceleration, the command signal to the "autopilot", taking into account the acceleration limit.

4) The autopilot and lateral missile dynamics are approximated (as a closed-loop system) by a first-order transfer function between the required and the actual lateral acceleration.

Lethality Model

A realistic lethality model of a missile warhead against an aircraft is a very complex one. It depends, for some given warhead features and aircraft vulnerability characteristics, on the relative terminal geometry of the interception (obtained from a six-degree-of-freedom simulation), as well as on the accurate point of the detonation determined by the proximity fuse. The warhead features—such as the number of fragments, their weight, and their velocity distribution—determine the lethal range against a well-defined area of the target.

In a point-mass model the only available information is the miss distance, and it is assumed that the warhead detonates at the point of closest approach. Moreover, the vulnerability of the target has to be taken to be uniform. Therefore, in this study the probability of destroying the target (P_d) is determined by a single-valued function of the actual miss distance, having only two parameters: the overall reliability of the guidance system $(P_d)_{\max}$ and a characteristic lethal radius R_ℓ. If the miss distance R_f is smaller than R_ℓ, then $P_d = (P_d)_{\max}$. For $R_f \geq R_\ell$, the following functional relationship was selected:

$$P_d(R_f) = (P_d)_{\max} \exp\left[-4\left(\frac{R_f}{R_e} - 1\right)^2\right] \qquad (3)$$

This model, replacing the crude on-off type model used in Ref. 9, expresses in a smooth form that warhead efficiency rapidly decays beyond the lethal radius.

In an imperfect information scenario, the miss distance is a random variable. For a given pair of pure strategies, δ_{ei} and δ_{pk}, the single shot kill probability (SSKP) is expressed by

$$P_{ik} = E\{P_d(R_f) \mid \delta_{ei}, \delta_{pk}\} \qquad (4)$$

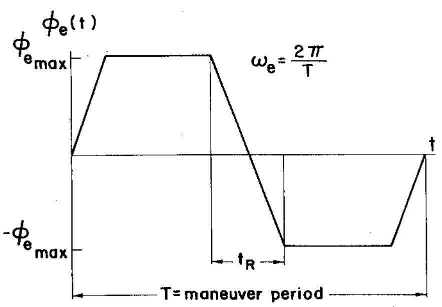

Fig. 3 Periodical target maneuver shape.

where the expectation is taken over the ensemble of all measurement noise samples, as well as the random phases of the target maneuver and the electronic jinking.

Performance Measures

The cost function of the game (to be maximized by the pursuer and minimized by the evader) is the overall SSKP of the missile

$$J = \Sigma\Sigma\alpha_i\beta_k P_{ik}, \quad \Sigma\alpha_i = \Sigma\beta_k = 1, \quad (\alpha_i \geq 0, \ \beta_k \geq 0) \qquad (5)$$

The solution of the game includes (for given Δ_e and Δ_p) the optimal mixed strategies α_i^* and β_j^* and the value of the game

$$\mathcal{V}_m = \min_{\alpha_i} \max_{\beta_j} J = J(\alpha_i^*, \beta_j^*) \qquad (6)$$

Obviously, $\mathcal{V}_m$, α_i^*, and β_j^* depend on the pure strategy sets Δ_e and Δ_p involved.

The objective of the guidance law designer is to find, against a *given* Δ_e, the best pure strategy set of the pursuer $\tilde{\Delta}_p$, i.e., how many and which pure guidance strategies δ_{pk} should be programmed into the missile, as well as the corresponding probability distribution β_k^* for selecting one of these strategies, so that

$$\mathcal{V}_m(\Delta_e, \tilde{\Delta}_p) \geq \mathcal{V}_m(\Delta_e, \Delta_p) \qquad (7)$$

for all admissible sets Δ_p.

In the guidance law synthesis the performance measure to be maximized is $\mathcal{V}_m$, the guaranteed (worst case) value of SSKP against an *optimal* mixed evader strategy $\alpha_i^*(\Delta_e, \tilde{\Delta}_p)$.

If the evader doesn't know $\tilde{\Delta}_p$, then α_i^* cannot be determined. In this case, the evader should use a *uniform* distribution on its pure strategy set. Against such evader policy, the appropriate performance measure is the *average* missile effectiveness, which is always higher than the "worst case."

In this paper, for comparison purposes, both the average SSKP and its guaranteed value $\mathcal{V}_m$ are presented.

Engagement Parameters

For the computational results presented in this paper several parameters of the previously described models were kept fixed. The numerical values of these parameters are listed in Table 1.

III. Synthesis of the Optimal Guidance Strategy

Mixed Strategy Guidance Law

MSG is the new approach for guidance law design.[9-14] It generates an optimal mixed strategy based on an optimal set of pure guidance strategies. Each of the pure guidance strategies

Table 1 Fixed engagement parameters

Initial conditions	
Initial range	$R_0 = 4500$ m
Initial pursuer heading	$\psi_{p0} = 0$ deg
Initial evader heading	$\psi_{e0} = 180$ deg
Evader parameters	
Velocity	$V_e = 300$ m/s
Lateral acceleration limit	$(a_e)_{\max} = 50$ m/s^2
Roll dynamics (Fig. 3)	$t_R = 2$ s
Glint parameters	
Band width	BW = 2 Hz
Standard deviations (in body axes)	$\sigma_x = 3.7$ m
	$\sigma_y = 2.5$ m
	$\sigma_z = 0.05$ m
Amplitude of electronic jinking	$w_{\max} = 4.7$ m
Pursuer parameters	
Velocity	$V_p = 600$ m/s
Lateral acceleration limit	$(a_p)_{\max} = 150$ m/s^2
Autopilot time constant	$\tau_p = 0.2$ s
Warhead lethality range	$R_\ell = 4.0$ m
Reliability factor	$(P_d)_{\max} = 0.9$

is composed of two elements: a "perfect information guidance law," developed on the basis of a linearized differential game model,[1,2] and an "estimator." The input for this guidance law, summarized in the Appendix, is the zero-effort miss distance, which is based on the LOS rate and on a compensation term for the missile's own lateral acceleration. It is independent of the evader acceleration, as implied by differential game theory. Since the scenario is noise corrupted, the actual input of the guidance law comes from the estimator in the form of a steady-state Kalman filter. Though assumptions on the evader's maneuver are incorporated in the dynamic model of the estimator, the estimated zero-effort miss distance does not include the evader acceleration. For a given structure, the estimator is determined by a set of parameters to be selected by the designer. In the present study the maximum dimension of such an estimator is either 4 or 6 depending on whether EJ is considered (see Fig. 3 in Ref. 13). The combination of each estimator and the perfect information guidance law forms a different "pure guidance strategy" δ_{pk}.

Design Parameters

In the estimator, the random target maneuver and the eventual EJ are considered as stochastic processes, approximated by a "shaping filter" fed by white noise.[17] In the present investigation, all shaping filters have the following form:

$$G_i = \frac{s+\gamma_i}{s^2+2\zeta_i\tilde{\omega}_i s+\tilde{\omega}_i^2}, \qquad i=e,j \tag{8}$$

Each shaping filter is fed by white noise of a given spectral density Φ_i $(i=e,j)$ where

$$\Phi_e = k_e(g\,n_{\max})^2 \tag{9}$$

$$\Phi_j = k_j(w_{\max})^2 \tag{10}$$

In addition to the two sets of four parameters (k_i, γ_i, ζ_i, and $\tilde{\omega}_i$), one also has to consider the spectral density of the glint Φ_{gl} which is the inherent measurement noise. Thus, the designer has (in the limitations imposed by the structure of the filter) a set of nine parameters, which constitute the assumptions made on the behavior of the evader. Based on these parameters, the gains of the Kalman filter are computed by solving the appropriate algebraic Riccati equation.

Computational Effort

To appreciate the computational effort involved in this design process, one has to remember that it is essentially an optimization problem in a high dimensional parameter space. Even if some of the parameters are kept fixed, the numerical workload is overwhelming. The computation of the end result, i.e., the SSKP, for any *given* set of parameters, involves a very large number of end-game simulations. For each evader strategy (i.e., a given combination of maneuver and electronic jinking options), 200 Monte Carlo simulations were used to obtain a reliable ensemble average of the outcome. Since the evader's pure strategy set consists of up to 90 elements, obtaining a reliable guaranteed SSKP value for any given set of estimator parameters (i.e., for each pure guidance strategy) involves 18,000 simulation runs.

The simulations were carried out on Technion's IBM 370/168 mainframe computer. The computation of a single value of P_{ik} (for a given pair of evader and pursuer strategies) based on the average of 200 Monte Carlo runs required about 2–3 min of CPU time. Therefore, the performance assessment of any pure guidance strategy (against the entire set of evader options) took between 3–4.5 h of CPU time.

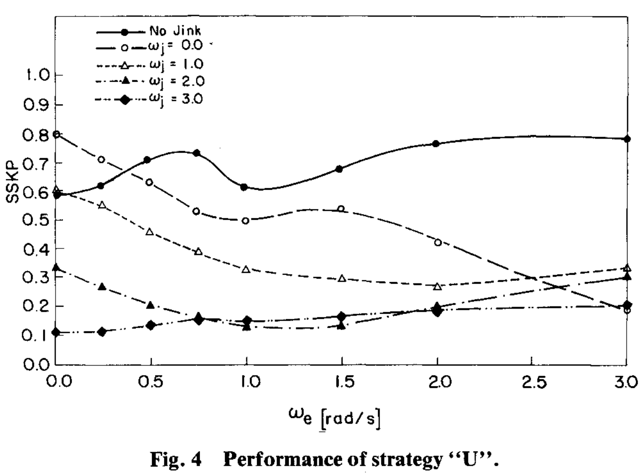

Fig. 4 Performance of strategy "U".

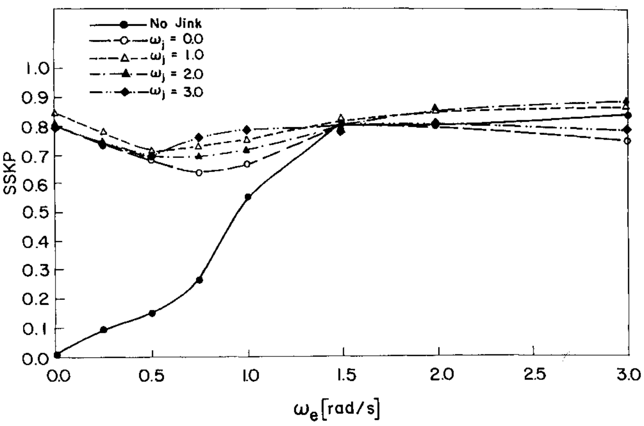

Fig. 5 Performance of strategy "J".

Simplified Design

The first exploratory design phase was based on the assumption that the missile can identify whether EJ is applied or not. This assumption allowed the optimization of two separate pure guidance strategies, one against a target with combined periodical random phase maneuvers and EJ and another against a randomly maneuvering target in an ECM-free (NJ) environment.

Against a target in an NJ scenario the optimal strategy, denoted "U", is characterized by $\Phi_{gl}=1.0$ and $k_j=0$ (strong glint effect), and the parameters are $k_e=0.1$, $\gamma_e=2.0$ (1/s), $\zeta_e=0.15$, and $\tilde{\omega}_e=0.1$ rad/s.

Whenever EJ was activated, a small value of $\Phi_{gl}=0.04$ was selected to characterize the fact (already mentioned earlier) that during EJ the effect of glint is negligible. (Zero value of Φ_{gl} would have created a singularity in the Kalman filter.) The other parameters of the shaping filter for this pursuer strategy, denoted "J", are $k_e=6.3$, $\gamma_e=1.8$ (1/s), $\zeta_e=0.151$, and $\tilde{\omega}_e=0.78$ rad/s, and $k_j=0.35$, $\gamma_j=38.0$ (1/s), $\zeta_j=0.155$, and $\tilde{\omega}_j=0.70$ rad/s. Each pure guidance strategy performed very well in its own environment as can be seen in Figs. 4 and 5. Strategy "U" guaranteed an SSKP of 0.59 against all periodic maneuvers with NJ, whereas strategy "J" provided a guaranteed SSKP of 0.65 against all 81 combinations of EJ and periodical maneuvers.

Most unfortunately, however, both strategies failed completely in the other environment. Against a constant target maneuver ($\omega_e=0$) with NJ, strategy "J" had a *zero* kill probability. At the other end, the performance of strategy "U" against a combined maneuver and EJ ($\omega_e=1$ rad/s and $\omega_j=3$ rad/s) was also very poor: an SSKP of only 0.11.

The optimal mixed strategy, denoted "M", based on these two pure strategies provided an SSKP of 0.35 in a combined EJ/NJ scenario.

Since some earlier results indicated that one can achieve an SSKP higher than 0.35, it was decided to search for optimal pursuer strategies against the entire evader strategy set of 90 elements.

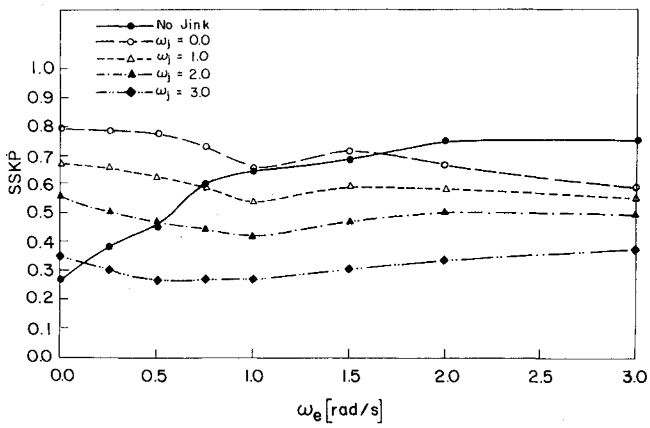

Fig. 6 Performance of strategy "F".

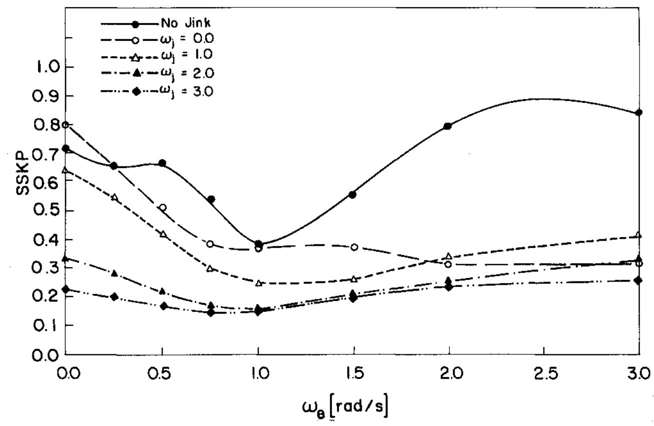

Fig. 7 Performance of strategy "L".

Design Phase I

In the first phase of the new design process, the search concentrated on the best pure guidance strategy. As the basis of the design $\Phi_{gl} = 1.0$ was selected. The optimal values of the other parameters were obtained as follows: $k_e = 8.92$, $\gamma_e = 0.572$ (1/s), $\zeta_e = 0.153$, and $\tilde{\omega}_e = 0.333$ rad/s and $k_j = 0.053$, $\gamma_j = 33.39$ (1/s), $\zeta_j = 0.172$, and $\tilde{\omega}_j = 0.426$ rad/s. The performance of this pure guidance strategy, denoted in the sequel as strategy "F", yielded a guaranteed SSKP of 0.27 as can be seen in Fig. 6. The evader can force this SSKP by choosing one of the following strategies: 1) constant maneuver with NJ or 2) maneuvers at frequencies ranging from 0.5 to 1 rad/s and EJ at 3 rad/s. In fact, the optimal mixed strategy of the evader against such a missile is some combination of the aforementioned evasive strategies. (The solution is not unique.)

Certainly, strategy "F" performs better in the minmax sense than strategy "J". This improvement can be attributed to the higher value of Φ_{gl}, the shift of the frequencies $\tilde{\omega}_e$ and $\tilde{\omega}_j$ to lower values, and the relative increase in the gain of the maneuver shaping filter with respect to the gain of the other one. This guidance strategy is, however, far from being optimal as a mixed strategy, as indicated by the results of the simplified design. The next step is, therefore, to search for a set of two pure guidance strategies that can be optimally mixed to maximize the SSKP.

Design Phase 2

The search process for the optimal set of parameters resulted in the following pair of pure optimal strategies. One of them turned out to be strategy "J" itself. To compensate for its weak point (constant target maneuver and NJ), the mixed strategy concept requires that the other element in this set of two should have very good performance against such evader strategy. The pure guidance strategy that was found to be a suitable partner of "J" is denoted "L" and has a shaping filter of the following parameters: $\Phi_{gl} = 1.0$, $k_j = 0$, $k_e = 0.10$, $\gamma_e = 0.5$ (1/s), $\zeta_e = 0.15$, and $\tilde{\omega}_e = 0.1$ rad/s. The weakness of this strategy, as can be seen in Fig. 7, is in an ECM scenario, in the region of $0.25 < \omega_e < 1.5$ rad/s, with $\omega_j \geq 2.0$ rad/s (where strategy "J" has good performance). It provides, however, an SSKP of 0.73 against a constant target maneuver and NJ.

An interesting observation can be obtained by comparing the performance of the guidance strategies "U" and "L", both in an NJ scenario, as shown in Fig. 8. From this comparison it is clear that "U" indeed performs better as a single strategy, having almost no weak points (its lowest SSKP is 0.59). The strategy "L" has, however, a better local performance at $\omega_e = 0$ (SSKP = 0.73) and its neighborhood (at the weak spot of strategy "J") but has a substantial performance degradation at $\omega_e = 1.0$ rad/s (with SSKP = 0.38). Note that the difference between the two guidance strategies is only in a single parameter: $\gamma_e = 0.5$ (1/s) in "L" and $\gamma_e = 2.0$ (1/s) in "U".

The optimal mixed strategy based on the two strategies "J" and "L" (0.55% of "L" and 0.45% of "J") is denoted "M*". It provides a guaranteed (worst case) SSKP of 0.4 (see Fig. 9). The weak points of "M*" are exactly the same as the weak points of strategy "F", but its performance at these points is about 50% better. The optimal mixed evader strategy against "M*" is, of course, the same as that against "F". Because of the overwhelming computational effort further improvements were not attempted, and the process of mixed guidance strategy synthesis was stopped.

It is easy to see that the mixed strategy "M*" is better than firing a salvo of two missiles, one with strategy "L" and the other with strategy "J", against each target, even in the case where the target randomly selects one of the "worst case"

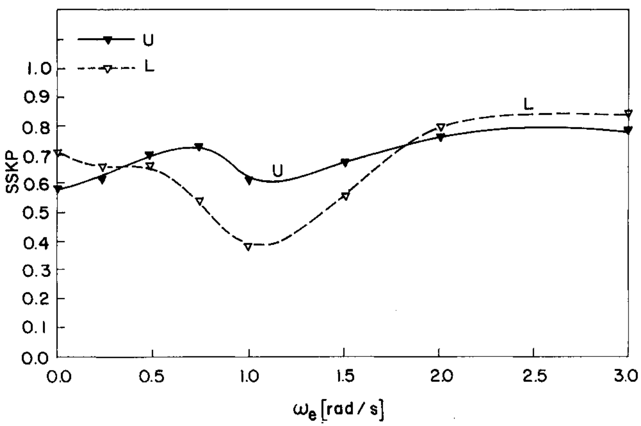

Fig. 8 Comparison of strategies "U" and "L" in an ECM-free scenario.

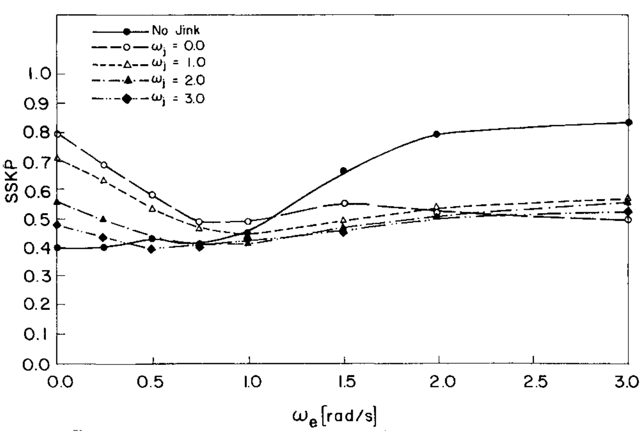

Fig. 9 Performance of strategy "M*".

Table 2 SSKP summary

Case	A[a]	B[b]	C[c]	D[d]	E[e]
PN	0.89	0.45	0.57	0.26	0.09
APN	0.80	0.47	0.66	0.26	0.075
U	0.86	0.49	0.71	0.29	0.11 (0.59)
J	0.87	0.85	0.59	0.79	0.00 (0.65)
M	0.86	0.58	0.64	0.47	0.35
F	0.84	0.65	0.66	0.53	0.27
L	0.88	0.49	0.67	0.29	0.13 (0.38)
M*	0.88	0.65	0.63	0.52	0.40

[a]Nonmaneuvering target, NJ.
[b]Nonmaneuvering target, with EJ (average).
[c]Maneuvering target, NJ (average).
[d]Maneuvering target, with EJ (average).
[e]Guaranteed (worst case) performance (∇_m).

evader strategies. If it performs constant maneuver without ECM, it will escape the "J" missile, but will be destroyed by the "L" missile with the probability of 0.73. A target using the worst combination of EJ and periodical maneuvering will be destroyed by the "J" missile with a probability of 0.7 and by the "L" missile with a probability of 0.13. The joint kill probability of the salvo is, therefore, $1-[(0.3)(0.87)]=0.74$. For any probability distribution between the two "worst case" maneuvers, the kill probability per missile of a salvo is between 0.365–0.37, lower than the guaranteed SSKP of "M*", which is 0.4.

Comparison

For the sake of completeness it seems worthwhile to provide a comparison between the different guidance strategies obtained by the MSG design and conventional guidance laws such as PN and APN.

The PN guidance law in the comparison had a constant "effective proportional navigation gain" $N'=3.0$, and the estimated LOS rate was obtained from the seeker via a first-order low-pass filter with a time constant of 0.3 s.

Also in the APN, which includes the estimate of the evader's lateral acceleration as a component of the estimated state, the guidance gain of $N'=3$ was selected. The estimator was a steady-state Kalman filter based on a "random telegraph" type (first-order) target maneuver model with the parameter $\lambda_t=0.5$ (1/s).

Both conventional guidance laws had very poor performance (SSKP less than 0.1) against some combinations of EJ and periodical target maneuver.

The comparison is presented in Table 2. The results for cases B, C, and D are averaged over the investigated range of maneuver and jinking frequencies (both between 0 to 3 rad/s) assuming a uniform distribution. The numbers in parentheses in column E indicate the guaranteed performance if the presence (or absence) of EJ can be identified.

The average values of SSKP have an important practical significance. If the evader has no information on the MSG parameters, it is rather unlikely that it will use only the worst case maneuvers. Against an ideal MSG design the optimal mixed evader strategy is a uniform probability distribution. The outcome of any other distribution is bounded between the previously defined average and the guaranteed SSKP values.

Note that the numerical results presented in this paper and summarized in Table 2 are for a single set of representative, but otherwise arbitrarily selected, parameters (given in Table 1). However, a recent sensitivity analysis[18] indicated that the main results remain qualitatively valid in a rather large domain of the parameter space.

IV. Conclusions

The results presented in this paper provide a convincing demonstration that the mixed strategy guidance approach, based on an imperfect information differential game formulation, yields a superior homing performance compared with any other known guidance law. Although the present study did not go beyond a pure strategy set of two elements for the pursuer, it demonstrated the substantial improvement that can be achieved over a single guidance strategy.

A very important conclusion derived from the investigation is its implication on aircraft survivability. Neither periodical maneuvering nor electronic jinking, if they are applied separately, can provide a satisfactory survival probability against advanced guided missiles. Their combination (randomly selected from a reasonably wide range of frequencies) has, however, a very strong synergistic effect on survivability enhancement.

From the guided missile designer's point of view, the results indicate that the more complete the strategy set of the evader against which the missile is designed, the higher the confidence in the guaranteed (worst case) homing performance. A device in the missile seeker that identifies the presence of electronic jinking can reduce the survivability enhancement resulting from randomly combined electronic jinking and periodical maneuvers.

Appendix: Perfect Information Guidance Law

This guidance law was derived in Ref. 1 based on a perfect information differential game formulation. The cost function of the game, to be minimized by the pursuer (the missile) and maximized by the evader, is the absolute value of the miss distance. Linearized kinematics in the vicinity of a "collision course" and constant velocities are assumed, implying a constant closing velocity and, as a consequence, a fixed final time. The evader has ideal dynamics, whereas pursuer dynamics are approximated by a first-order time constant τ_p. Both players have hard constraints on the lateral acceleration. The ratio of these maneuver constraints

$$\mu \triangleq \frac{(a_p)_{\max}}{(a_e)_{\max}} \tag{A1}$$

is the parameter governing the game solution. The critical normalized time to go of the game Θ_c is determined by the solution of the following implicit equation:

$$\Theta_c^2(1-1/\mu) = 2\left[\Theta_c - 1 + \exp(-\Theta_c)\right] \tag{A2}$$

For $\Theta > \Theta_c$, the commanded lateral acceleration of the missile $(a_p)^c$ is a linear function of the zero effort miss Z, subject to saturation

$$(a_p)^c = \frac{K(\Theta,\mu)}{\tau_p^2\Theta^2}Z \tag{A3}$$

where

$$Z = y + \dot{y}\tau_p\Theta - a_p\tau_p^2\left[\Theta - 1 + \exp(\Theta)\right] \tag{A4}$$

and $K(\Theta,\mu)$ is the guidance gain expressed by

$$K(\Theta,\mu) = \frac{2\Theta^2}{(1-1/\mu)\Theta^2 - 2\left[\Theta - 1 + \exp(\Theta)\right]} \tag{A5}$$

For $\Theta \le \Theta_c$, the commanded acceleration is on the constraint

$$(a_p)^c = (a_p)_{\max}\,\mathrm{sign}\{Z\} \tag{A6}$$

In an imperfect information scenario, the values of y and $\dot{y}$ are to be replaced by the estimates $\hat{y}$ and $\hat{\dot{y}}$.

Acknowledgment

The work described in this paper was sponsored by the Air Force Office of Scientific Research, Grant 89-0528.

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